Towards Automated Load Balancing via Spectrum Slicing for FEAST-like Solvers

30.11.2016 | Jan Winkelmann, Edoardo Di Napoli
The Hermitian Interior Eigenvalue Problem

- Let \( H \in \mathbb{C}^{n \times n} \) be Hermitian
- Find all \((\lambda, v)\) with \( \lambda \in [a, b] \), such that
  \[
  H v = \lambda v
  \]

- All solvers are sensitive to the spectrum \( \{ \lambda \mid H v = \lambda v \} \)
- Knowledge about the spectrum may increase performance
- Today: Load balancing for FEAST-like eigensolvers
Our goal is to combine

1. The ability to design filters with specific properties
2. Cheaply available information on the spectrum

In order to achieve good parallel performance on FEAST-like solvers.
FEAST: A Rationally Filtered Interior Eigensolver

**FEAST** (Matrix $A$, Approx $Y$)

```
while $||Y^*AY - \Lambda|| < \text{tol}$ do
  $U \leftarrow f(A)Y$
  $U \leftarrow QR(U)$
  $B \leftarrow U^*AU$
  Solve: $\Lambda = W^*BW$
  $Y \leftarrow UW$
return $(\Lambda, Y)$
```

$$f(A) = \sum_{i=1}^{p} \alpha_i(lz_i - A)^{-1}$$
Levels of Parallelism in FEAST

1. Partition the search interval into multiple, independent intervals
2. $p$ Linear System Solves per iteration

\[ U := f(A)Y = \sum_{i=1}^{p} \alpha_i (Iz_i - A)^{-1} Y \]

3. Linear System Solve

\[ (A - Iz_i)U = \alpha_i Y \]
State of the Art: Approximate Eigenvalue Counts

-2 -1.5 -1 -0.5 0 0.5 1 1.5 2 2.5 3

Eigenvalue

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![Eigenvalue cartoon of eigenvalue count](cite edo, sakurai)
State of the Art: Approximate Eigenvalue Counts

- 1.5
- 1
- 0.5
- 0
- 0.5
- 2  -1.5  -1  -0.5  0  0.5  1  1.5  2  2.5  3

= 10  = 10  = 10

Eigenvalue

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NLLS filters: Optimized filters for FEAST

Non-Linear Least-Squares optimized filters:

\[
\min_{\alpha_i, z_i} \int_{-\infty}^{\infty} w(t) \left| h(t) - \sum_{i=1}^{n} \frac{\alpha_i}{t - z_i} \right|^2 dt
\]

- Good results when compared to existing filters
- Ability to 'design' filters for special use-case
- Forthcoming publication in beginning of 2017
Future Work: Load Balancing with NNLS Filters and DoS
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- Eigenvalue
- Elliptical Filter ($p = 4$)
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Opportunities for Collaboration

Cheap Information on the Spectrum

- Given Matrix $A$, sparse/dense, (non-)Hermitian
- Obtain information on the spectrum of $A$ around $[a, b]$
- As cheaply as possible (Trace estimation, Lanczos)

Eigenproblems with Difficulties in Load Balancing

- Preferably Hermitian problems
- Suspected potential for better Load Balancing
- Ideally with FEAST-like solvers
The End

Thank you for your attention
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Future Work: Load Balancing with Krylov-Solvers

The Linear System Solve:

\[(A - Iz_i)U = \alpha_i Y \quad \forall 1 \leq i \leq p\]

- For large \(p\), \(\min_{1 \leq i \leq p} |\Im(z_i)|\) becomes very small
- Then \((A - Iz_i)\) may become nearly singular
- Workload imbalance with Krylov-based solvers

Another aspect of load balancing

- We can influence the imaginary part of \(z_i\)
- Reduces Load Imbalance (at cost of convergence speed)
Condition Number of \((A - Iz_i)\)

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Legend:
- NNLS
- Elliptical
- Gauss
Performance Profile

![Graph showing performance profile with two lines, one for Gauss and one for NNLS, indicating the problems solved against the performance ratio.]

Problems solved vs. Performance Ratio for Gauss and NNLS methods.